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Absorption by double-centered D3-branes and the Coulomb branch of $\mathcal{N} = 4$ SYM theory

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Abstract: We calculate the classical cross-section for absorption of a minimally coupled scalar in the double-centered D3-brane geometry. The dual field theory has gauge symmetry broken to $\mathrm{S}(\mathrm{U}(N_1)\times\mathrm{U}(N_2))$ and is on the Coulomb branch of $\mathcal{N} = 4$ Super Yang-Mills theory. Our analysis is valid at energy scales much smaller than the W particles mass, giving logarithmic corrections to the cross-section calculated at the IR conformal fixed points. These corrections are associated with deformations of the $\mathcal{N} = 4$ Super Yang-Mills theory by irrelevant operators that break conformal invariance and correspond to processes where a virtual pair of gauge particles or a virtual pair of W bosons interact with the incident wave to create a pair of gauge particles.

Keywords: D-branes, Black Holes in String Theory, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

The famous AdS/CFT duality [1, 2, 3] has been investigated intensively over the past two years (see [4] for a review and an exhaustive list of references). The most studied case is the conjectured equivalence between type-IIB string theory on $AdS_5 \times S^5$ and the low energy theory on N D3-branes given by $\mathcal{N} = 4$ Super Yang-Mills (SYM) theory with SU(N) gauge group. Unfortunately, due to our limited knowledge of Ramond-Ramond backgrounds in string theory, we are bound to consider type-IIB supergravity on $AdS_5 \times S^5$, which is dual to large N SYM theory at strong coupling.

The moduli space of SU(N) $\mathcal{N} = 4$ SYM theory is $\mathbb{R}^{6(N-1)}/S_N$, parameterizing the relative position of N identical D3-branes in the transverse space $\mathbb{E}^6$. At the origin of the moduli space the theory is superconformally invariant. This fact is related to symmetry enhancement at the D3-brane near-horizon geometry [5] and it guarantees that both theories are invariant under the PSU$(2, 2|4)$ supergroup. However, many interesting theories like QCD are not conformally invariant. The main purpose of this paper is to study the AdS/CFT duality when conformal invariance is broken by separating the D3-branes. In particular, we consider the case of two parallel stacks of D3-branes. In the dual field theory, moving away from the origin of the moduli space means that the scalar fields have non-zero expectation values. This is also known as the Coulomb branch of the theory. The dual gravitational configuration is related to the double-centered type-IIB D3-brane background. Previous work on the Coulomb branch of the correspondence can be found in [6]–[17]. Other studies of the duality with broken conformal invariance are [18]–[27]. Consider $N = N_1 + N_2$ D3-branes described at low energies by the SU(N) $\mathcal{N} = 4$ SYM theory. Next, Higgs the system by giving an expectation value to the scalar fields according to

$$\frac{\langle \vec{Y} \rangle}{2\pi\alpha'} \equiv \langle \vec{\phi} \rangle = \frac{\vec{\Delta}}{2\pi\alpha'} \begin{pmatrix} I_1 & 0 \\ 0 & -I_2 \end{pmatrix}, \tag{1.1}$$

where I_i is the $N_i \times N_i$ unit matrix. The N_1 and N_2 D3-branes are separated by a distance 2Δ and the gauge symmetry is broken to S(U(N_1)$\times$U(N_2)). Furthermore, the theory is no longer conformally invariant and there is a scale naturally set by the W particles mass that arises from the symmetry breaking process:

$$m_W = \frac{\Delta}{\pi\alpha'} \, . \tag{1.2}$$

The theory is still maximally supersymmetric, i.e. it has 16 conserved supercharges (but it is not invariant under the extra 16 superconformal charges). Hence, we shall be interested in maximally supersymmetric RG flows [24]. In the UV, for energies $\epsilon \gg m_W$, we recover the conformal invariant SU(N) theory, while for $\epsilon \ll m_W$ the theory flows to the IR conformal fixed points with SU(N_i) gauge symmetry. In particular, the gauge coupling beta function vanishes along such flows and the theory can be described starting from a IR fixed point by a deformation given by a non-renormalizable, irrelevant operator with scaling dimension 8, as was conjectured in [18, 19, 24].

We shall be able to test the above conjecture by studying the low energy scattering of the dilaton field (or more generally of a minimally coupled scalar) in the dual double-centered D3-brane geometry. To be more precise, we find logarithmic corrections to the cross-section calculated at the IR conformal fixed points. These corrections are associated with processes where a pair of gauge particles is created via the interaction of the incident dilaton wave with a virtual pair of gauge particles or a virtual pair of W bosons. Hence, even at energies $\epsilon \ll m_W$, well below the threshold for W boson pair production, the dual gravity theory "knows" about the W particles. This result provides further evidence for the duality to hold even when conformal invariance is broken, as is the case in the Coulomb branch of the field theory away from the origin of the moduli space of vacua.

The paper is organized as follows: In section 2 we introduce the prolate spherical coordinates, which are very useful to solve scattering problems in a double-centered black hole background [28]. Then, we analyze the double-centered D3-brane geometry in the decoupling limit. In section 3 we calculate the classical cross-section for absorption of a minimally coupled scalar in the double-centered D3-brane background by using the improved matching technique developed in [18]. These results are interpreted from the dual field theory point of view in section 4. Finally, we give some concluding remarks in section 5.

2. Prolate spherical coordinates and decoupling limit

In this section we analyze the double-centered D3-brane geometry and the corresponding decoupling limit using prolate spherical coordinates [28]. The D3-brane geometry is described by the metric

$$ds^2 = H^{-1/2}\, ds^2\left(\mathbb{M}^4\right) + H^{1/2}\, ds^2\left(\mathbb{E}^6\right). \tag{2.1}$$

In the double-centered case the harmonic function H is given by

$$H = 1 + \frac{{R_1}^4}{|\vec{y} - \vec{\Delta}|^4} + \frac{{R_2}^4}{|\vec{y} + \vec{\Delta}|^4}\,, \tag{2.2}$$

where $\vec{y}$ are the coordinates in the space orthogonal to the branes $\mathbb{E}^6$ and

$${R_i}^4 = 4\,\pi\alpha'^2 g_s N_i\,. \tag{2.3}$$

This solution is associated with a system of N_1 D3-branes placed at $\vec{\Delta}$ and N_2 D3-branes at $-\vec{\Delta}$.

Next, we place the branes along the y^6 axis and change to prolate spherical coordinates

$$y_6 = r\cos\theta_1 \equiv \Delta\cosh\Psi\cos\theta\,, \qquad \rho = r\sin\theta_1 \equiv \Delta\sinh\Psi\sin\theta\,, \tag{2.4}$$

where $(r, \theta_1, \theta_2, \ldots, \theta_5)$ are the usual spherical coordinates and $(\Psi, \theta, \theta_2, \ldots, \theta_5)$ are the prolate spherical coordinates. Of course, the system has SO(5) invariance and the transformation $(r, \theta_1) \to (\Psi, \theta)$ is given by

$$r^2 \equiv \Delta^2\left(\cosh^2\Psi\cos^2\theta + \sinh^2\Psi\sin^2\theta\right), \qquad \tan\theta_1 \equiv \tanh\Psi\tan\theta\,. \tag{2.5}$$

It is convenient to define the (η, μ) coordinates by

$$\eta = \cosh\Psi\,, \qquad \mu = \cos\theta\,, \tag{2.6}$$

with $\eta \in]1, +\infty[$ and $\mu \in [-1, 1]$. For $\eta \gg 1$, we have $\eta \sim r\Delta$ and $\theta \sim \theta_1$, i.e. the spherical coordinates are recovered. The coordinate system is singular at $\eta = 1$ (and μ arbitrary). In prolate spherical coordinates the harmonic function H becomes

$$H = 1 + \frac{{R_1}^4}{\Delta^4(\eta - \mu)^4} + \frac{{R_2}^4}{\Delta^4(\eta + \mu)^4}\,, \tag{2.7}$$

and the metric element on the transverse space $\mathbb{E}^6$

$$ds^2\left(\mathbb{E}^6\right) = \Delta^2\left[\left(\eta^2-\mu^2\right)\left(\frac{d\eta^2}{\eta^2-1}+\frac{d\mu^2}{1-\mu^2}\right)+\left(\eta^2-1\right)\left(1-\mu^2\right)d\Omega_4{}^2\right]. \tag{2.8}$$

In the limit $\eta \gg 1$ we have the asymptotics of the $N = N_1 + N_2$ charged D3-brane solution.

We proceed by analyzing the geometry down the throats. Consider the limit $\eta \to 1$ and $\mu \to 1$ (or -1). This corresponds to the geometry near the horizon of the first (second) stack of N_1 (N_2) branes. It is convenient to perform the following change of coordinates

$$\eta = 1 + a\,, \qquad \mu = \pm 1 \mp b\,. \tag{2.9}$$

Next, define the radial coordinate from the center of the i-th throat ζ_i and the angle with the $\mu = \pm 1$ direction φ_i by

$$\zeta_i = a + b\,, \qquad \cos\varphi_i = \frac{a-b}{a+b}\,. \tag{2.10}$$

To leading order in a and b (i.e. ζ_i) the spacetime metric reads

$$ds^2 = H^{-1/2}\,ds^2\left(\mathbb{M}^4\right) + H^{1/2}\left(d\zeta_i{}^2 + \zeta_i{}^2\,d\Omega_5{}^2\right) \tag{2.11}$$

with

$$H = 1 + \left(\frac{R_j}{2\Delta}\right)^4 + \left(\frac{R_i}{\Delta\zeta_i}\right)^4. \tag{2.12}$$

Note that here and henceforth we have $i \neq j$. Of course, for $\zeta_i \ll 1$ far down the throat the constant terms in the harmonic function H become irrelevant. Then, the geometry becomes the well known $AdS_5 \times S^5$ space with radius R_i.

To analyze the geometry in the decoupling limit, first consider the double-centered D3-brane solution. This is an asymptotically flat space with total charge $N = N_1 + N_2$ and with two throats centered at $\pm\vec{\Delta}$ (see figure 1a). In the previous paragraph we saw that the throat geometries are spherically symmetric and that far down the throats we obtain the $AdS_5 \times S^5$ single-centered black hole near-horizon geometry. The decoupling limit is defined by sending $\alpha' \to 0$ such that

$$\vec{\mathrm{U}} \equiv \frac{\vec{y}}{\alpha'}\,, \qquad \vec{D} \equiv \frac{\vec{\Delta}}{\alpha'}\,, \tag{2.13}$$

are kept fixed, as well as the dimensionless parameters g_s and N_i (we require $g_s \ll 1$ and $g_s N_i \gg 1$ for the supergravity approximation to hold). The spacetime metric becomes

$$\begin{aligned}\frac{ds^2}{\alpha'} = {} & \frac{1}{\sqrt{4\pi g_s}}\left(\frac{N_1}{D^4(\eta-\mu)^4}+\frac{N_2}{D^4(\eta+\mu)^4}\right)^{-1/2} ds^2\left(\mathbb{M}^4\right) + \\ & + \sqrt{4\pi g_s}\left(\frac{N_1}{D^4(\eta-\mu)^4}+\frac{N_2}{D^4(\eta+\mu)^4}\right)^{1/2} ds^2\left(\mathbb{E}^6\right), \end{aligned} \tag{2.14}$$

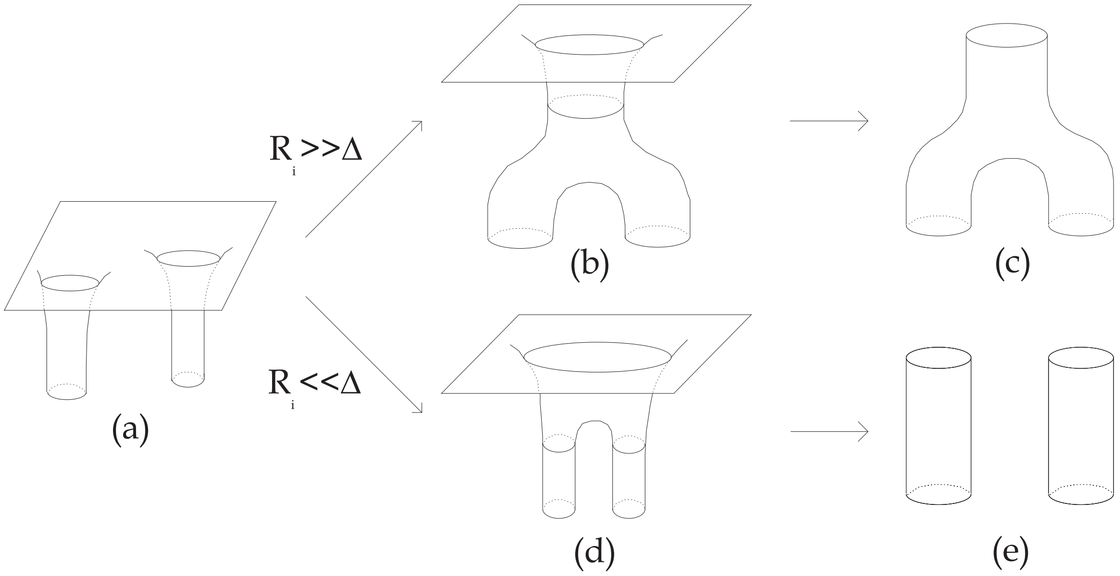

Figure 1: The different decoupling limits. The case $(a)\to(b)\to(c)$ corresponds to keeping Δ/α' fixed, while the case $(a)\to(d)\to(e)$ to keeping Δ finite.

where $ds^2(\mathbb{E}^6)$ is given by (2.8) with Δ^2 replaced by D^2. As we take the decoupling limit the throats' separation goes to zero and we have $R_i \gg \Delta$ (see figure 1b). In the limit we obtain the tree-like geometry sketched in figure 1c [29]. This geometry is dual to the Coulomb branch of $\mathcal{N}=4$ SYM theory as described in the introduction. In the limit $\eta \gg 1$, the geometry approaches $AdS_5 \times S^5$ near its boundary with radius $R = R_1 + R_2$. This corresponds to the conformal UV limit of the SU(N) field theory. In the other limits $\eta \to 1$, $\mu \to \pm 1$ we obtain the $AdS_5 \times S^5$ geometry with radius R_i. In the field theory side these correspond to the SU(N_i) IR conformal fixed points. At low energies $\epsilon \ll m_W$, we can start from the IR theory and consider some deformation of the field theory. This means that the geometry (2.14) is described by the throat metric (2.11) with the harmonic function H given by

$$H = \left(\frac{R_j}{2\Delta}\right)^4 + \left(\frac{R_i}{\Delta\zeta_i}\right)^4 . \tag{2.15}$$

The constant term is due to the other throat and will be interpreted in the field theory side as a result of integrating out the massive W particles.

A different possibility is to take the decoupling limit (2.13) with Δ finite and non zero. This means that $R_i \ll \Delta$ and that in the limit we have $\Delta/\alpha' \to \infty$. In this case, as we take the decoupling limit both throats are kept apart (figure 1d) and in the limit we obtain two disconnected $AdS_5 \times S^5$ spaces (figure 1e). In the low energy limit $\epsilon \ll m_W$, we shall study the effects in the dual field theory that correspond to connecting the two throats to asymptotically flat space as described by figure 1d.

3. Scattering of minimally coupled scalar

In this section we shall consider the problem of scattering a minimally coupled scalar in the double-centered D3-brane background. We assume that the incident wave is isotropic with frequency ω, i.e.

$$\phi = e^{-i\omega t}\,\phi(\eta,\mu)\,, \tag{3.1}$$

with $\phi(\eta,\mu) \sim \phi(\eta)$ for $\eta \gg 1$. In prolate spherical coordinates the Laplace equation reads

$$\left[\nabla^2_{(\eta,\mu)} + (\omega\Delta)^2 \left(1 + \frac{{R_1}^4}{\Delta^4(\eta-\mu)^4} + \frac{{R_2}^4}{\Delta^4(\eta+\mu)^4}\right)\right]\phi(\eta,\mu) = 0\,, \tag{3.2}$$

where

$$\nabla^2_{(\eta,\mu)} = \frac{1}{\eta^2-\mu^2}\left[\frac{1}{(\eta^2-1)^{3/2}}\,\partial_\eta\left(\left(\eta^2-1\right)^{5/2}\partial_\eta\right) + \right. \\ \left. + \frac{1}{(1-\mu^2)^{3/2}}\,\partial_\mu\left(\left(1-\mu^2\right)^{5/2}\partial_\mu\right)\right]. \tag{3.3}$$

To solve this equation we use the by now well-known matching technique. Start by splitting spacetime in the following four regions: near-horizon regions $\mathbf{I_i}$ for $\eta \to 1$ and $\mu \to \pm 1$; intermediate region **II** for $\omega = 0$; far region **III** for $\eta \gg 1$. The different

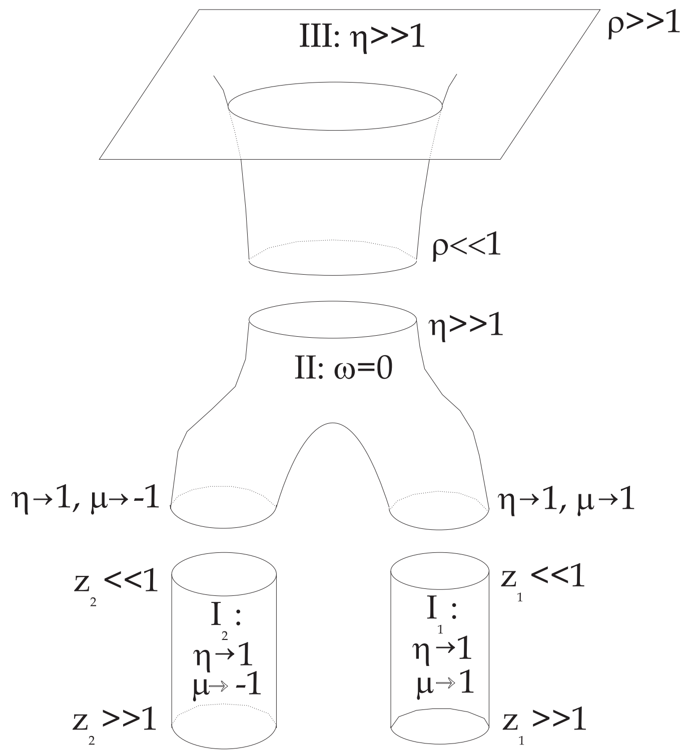

Figure 2: The different regions used in the solution matching technique.

regions are shown in figure 2. The differential equation (3.2) becomes in each region

$$\mathbf{I_i}:\left[\frac{1}{{\zeta_i}^5}\,\partial_{\zeta_i}\left({\zeta_i}^5\partial_{\zeta_i}\right)+(\omega\Delta)^2\left(1+\left(\frac{R_j}{2\Delta}\right)^4+\left(\frac{R_i}{\Delta\zeta_i}\right)^4\right)\right]\phi(\zeta_i)=0\,, \tag{3.4}$$

$$\mathbf{II}:\nabla^2_{(\eta,\mu)}\,\phi(\eta,\mu)=0\,, \tag{3.5}$$

$$\mathbf{III}:\left[\frac{1}{\eta^5}\,\partial_\eta\left(\eta^5\partial_\eta\right)+(\omega\Delta)^2\left(1+\frac{{R_1}^4+{R_2}^4}{(\Delta\eta)^4}\right)\right]\phi(\eta)=0\,. \tag{3.6}$$

Notice that the differential eq. (3.2) separates in the four regions. In region **III** we assumed that the dilaton wave is constant along S^5 because we consider a spherically symmetric incident wave. Also, in the throat regions $\mathbf{I_i}$ we assume spherical symmetry, i.e. there is no φ_i dependence. The reason is that the higher harmonic dependence in φ_i is related to higher partial waves absorption by each hole whose cross-section is smaller by a factor of $(\omega R_i)^4$ in comparison to the leading order term here calculated. The use of the $\omega=0$ equation in region **II** means that we are scattering the black holes at very low energy. In particular, it means that $\omega\Delta\ll 1$, i.e. the wave length of the incident wave is much larger than the holes separation distance.

First consider the throat regions $\mathbf{I_i}$. Defining the coordinate $z_i=\omega{R_i}^2/\Delta\zeta_i$ and setting $\phi^{I_i}(z_i)={z_i}^4\Psi^{I_i}(z_i)$, eq. (3.4) reads

$$\left[\frac{1}{{z_i}^5}\,\partial_{z_i}\left({z_i}^5\partial_{z_i}\right)+1+\frac{(\omega R_i)^4}{{z_i}^4}\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\right]\Psi^{I_i}(z_i)=0\,. \tag{3.7}$$

For ${z_i}^2\gg(\omega R_i)^4(1+(R_j/2\Delta)^4)$ this can be solved in terms of Bessel functions of order 2. The result is

$$\phi^{I_i}(z_i)={z_i}^2\left(A_i\,J_2(z_i)+i\,B_i\,N_2(z_i)\right), \tag{3.8}$$

where A_i and B_i are undetermined constants. In the fluxes method for the absorption cross-section calculation we require purely infalling flux at the horizons. Setting $A_i=B_i=1$ the ingoing flux at the horizons is $\mathcal{F}_{in}(H_i)\cong(\omega{R_i}^2)^4/\pi$. Notice that the validity of the approximation in regions $\mathbf{I_i}$ requires $\zeta_i\ll 1$ and ${z_i}^2\gg(\omega R_i)^4(1+(R_j/2\Delta)^4)$. It is not difficult to see that these conditions are compatible. To match this solution to the region **II** we require $z_i\ll 1$ and use the small argument expansion for the Bessel functions. Consistency with the above conditions requires

$$(\omega R_i)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\ll 1 \qquad \text{and} \qquad \frac{\omega{R_i}^2}{\Delta}\ll 1\,. \tag{3.9}$$

As usual we have $(\omega R_i) \ll 1$ and the condition $\omega R_i R_j/\Delta \ll 1$ holds for $\omega/m_W \ll 1$.[1] In the limit $z_i \ll 1$, (3.8) becomes

$$\phi^{I_i} \sim -\frac{4i}{\pi}\,. \tag{3.10}$$

In region **II**, the solution to the differential eq. (3.5) can be written in terms of the Gegenbauer polynomials G_l^2 as

$$\phi^{II}(\eta,\mu) = \sum_{l=0}^{\infty} C_l\; G_l^2(\eta)\; G_l^2(\mu)\,. \tag{3.11}$$

However, to match to the asymptotic behavior (3.10), only the constant term survives

$$\phi^{II} = C = -\frac{4i}{\pi}\,. \tag{3.12}$$

Finally, consider the asymptotic flat region **III**. Defining the coordinate $\rho = \omega\Delta\eta$ the corresponding differential equation becomes

$$\left[\frac{1}{\rho^5}\,\partial_\rho\left(\rho^5\partial_\rho\right) + 1 + \omega^4\frac{{R_1}^4 + {R_2}^4}{\rho^4}\right]\phi^{III}(\rho) = 0\,. \tag{3.13}$$

For $\rho^2 \gg (\omega R_i)^4$ the last term is negligible and again this equation can be solved in terms of Bessel functions of order 2. The result is

$$\phi^{III}(\rho) = \frac{1}{\rho^2}\left(DJ_2(\rho) + iEN_2(\rho)\right), \tag{3.14}$$

where D and E are undetermined constants. The validity of the approximation in region **III** requires $\eta \gg 1$ and $\rho^2 \gg (\omega R_i)^4$. These conditions are compatible because we are assuming that $(\omega {R_i}^2/\Delta) \ll 1$. As usual, in the limit $(\omega R_i) \ll 1$ we can set $\rho \ll 1$ and use the small argument expansion of the Bessel functions to match the solution to region **II**. Notice that the condition $\eta \gg 1$ is compatible with $\rho = \omega\Delta\eta \ll 1$ because $\omega\Delta$ is small. In the limit $\rho \ll 1$, (3.14) becomes

$$\phi^{III} \sim \frac{D}{8} = -\frac{4i}{\pi}\,, \tag{3.15}$$

where we matched the solution to ϕ^{II}. Then, the incoming flux at infinity is seen to be $\mathcal{F}_{in}(\infty) \cong (16)^2/(\pi^3\omega^4)$.

The probability for absorption by the i-th hole can be calculated as the ratio of the incoming flux at the i-th horizon to the incoming flux at infinity:

$$\mathcal{P}_i = \frac{\mathcal{F}_{in}(H_i)}{\mathcal{F}_{in}(\infty)} = \frac{\pi^2}{(16)^2}\,(\omega R_i)^8\,, \tag{3.16}$$

which is the same result as first obtained by Klebanov in the single-centered case [30]. The absorption cross-section is related to this probability by $\sigma_{\rm abs} = (32\pi^2/\omega^5)\mathcal{P}$.

[1] To be more precise, the condition $\omega R_i R_j/\Delta \ll 1$ is equivalent to $\omega \ll m_g$, where $m_g = m_W/\sqrt{gN}$ is the gravity mass gap. Hence, the domain of validity of this calculation is restricted to energies $\epsilon \ll m_g \ll m_W$. We shall comment on a possible field theory interpretation of this fact later.

3.1 Logarithmic corrections to absorption cross-section

The above calculation provides the leading term in an expansion of the cross-section in powers of (ωR_i) and $(\omega R_i\, R_j/\Delta)$. In this subsection we shall be more careful with the matching conditions across the different regions and determine the first corrections to the absorption probability (3.16) using the improved matching technique developed in [18]. At this point, those concerned with the dual field theory interpretation of our results may wish to skip directly to the final answer (3.44).

First consider the near horizon regions $\mathbf{I_i}$. Instead of dropping the last term in eq. (3.7) we solve this equation by expanding $\Psi^{I_i}(z_i)$ in powers of $(\omega R_i)^4(1+(R_j/2\Delta)^4)$:

$$\Psi^{I_i}(z_i) = \Psi_0^{I_i}(z_i) + (\omega R_i)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\Psi_1^{I_i}(z_i) + \cdots . \tag{3.17}$$

Writing eq. (3.7) order by order in the expansion parameter we have

$$\left[\frac{1}{{z_i}^5}\,\partial_{z_i}\left({z_i}^5\partial_{z_i}\right)+1\right]\Psi_0^{I_i}(z_i) = 0\,, \tag{3.18}$$

$$\left[\frac{1}{{z_i}^5}\,\partial_{z_i}\left({z_i}^5\partial_{z_i}\right)+1\right]\Psi_1^{I_i}(z_i) = -\frac{1}{{z_i}^4}\,\Psi_0^{I_i}(z_i)\,. \tag{3.19}$$

The solutions to the homogeneous equation are

$$\begin{aligned}\frac{J_2(z_i)}{{z_i}^2} &\sim \frac{1}{8}\left(1-\frac{{z_i}^2}{12}+\mathcal{O}\left({z_i}^4\right)\right),\\ \frac{N_2(z_i)}{{z_i}^2} &\sim -\frac{4}{\pi {z_i}^4}\left(1+\frac{{z_i}^2}{4}+\mathcal{O}\left({z_i}^4\log z_i\right)\right).\end{aligned} \tag{3.20}$$

In the previous calculation we found that $\Psi_0^{I_i} = A_i(J_2+iN_2)/{z_i}^2 \equiv A_i H_2^{(1)}/{z_i}^2$. The solution to the inhomogeneous equation for $\Psi_1^{I_i}$ can be written in an integral form

$$\Psi_1^{I_i}(z_i) = -\frac{\pi}{2{z_i}^2}\int^{z_i}\frac{d\sigma}{\sigma}\Psi_0^{I_i}(\sigma)\left(J_2(\sigma)N_2(z_i)-N_2(\sigma)J_2(z_i)\right), \tag{3.21}$$

to which a solution of the homogeneous equation can be added. This ambiguity is resolved by requiring purely infalling flux at the horizon and by matching to the solution in region **II**. The improved solutions in regions $\mathbf{I_i}$ are given by

$$\begin{aligned}\frac{1}{A_i}\,\phi^{I_i}(z_i) = {z_i}^2 H_2^{(1)}(z_i) - (\omega R_i)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\frac{\pi {z_i}^2}{2}\,\times\\ \times\int^{z_i}\frac{d\sigma}{\sigma^3}\,H_2^{(1)}(\sigma)\left(J_2(\sigma)N_2(z_i)-N_2(\sigma)J_2(z_i)\right).\end{aligned} \tag{3.22}$$

For small z_i this gives

$$\frac{1}{A_i}\,\phi^{I_i}(z_i) \sim -\frac{4i}{\pi}\left[1+\frac{{z_i}^2}{4}-(\omega R_i)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\times\right.$$
$$\left.\times\,\frac{1}{12{z_i}^2}\left(1-\frac{{z_i}^2\log z_i}{2}\right)+\cdots\right]. \qquad (3.23)$$

Next we consider the intermediate region **II**. The equation for $\phi(\eta,\mu)$ reads

$$\left[\nabla^2_{(\eta,\mu)}+(\omega\Delta)^2\left(1+\frac{{R_1}^4}{\Delta^4(\eta-\mu)^4}+\frac{{R_2}^4}{\Delta^4(\eta+\mu)^4}\right)\right]\phi(\eta,\mu)=0 \qquad (3.24)$$

and in the previous calculation we set $\omega=0$. Now we expand the solution in powers of $(\omega\Delta)^2$:

$$\phi^{II}(\eta,\mu)=\phi_0^{II}(\eta,\mu)+(\omega\Delta)^2\phi_1^{II}(\eta,\mu)+(\omega\Delta)^4\phi_2^{II}(\eta,\mu)+\cdots \qquad (3.25)$$

which gives order by order

$$\nabla^2_{(\eta,\mu)}\,\phi_0^{II}(\eta,\mu)=0\,, \qquad (3.26)$$

$$\nabla^2_{(\eta,\mu)}\,\phi_1^{II}(\eta,\mu)=-\left(1+\frac{{R_1}^4}{\Delta^4(\eta-\mu)^4}+\frac{{R_2}^4}{\Delta^4(\eta+\mu)^4}\right)\phi_0^{II}(\eta,\mu)\,, \qquad (3.27)$$

$$\nabla^2_{(\eta,\mu)}\,\phi_2^{II}(\eta,\mu)=-\left(1+\frac{{R_1}^4}{\Delta^4(\eta-\mu)^4}+\frac{{R_2}^4}{\Delta^4(\eta+\mu)^4}\right)\phi_1^{II}(\eta,\mu)\,. \qquad (3.28)$$

We saw before that $\phi_0^{II}=C$. The other solutions to the homogeneous equation in the summation (3.11) give contributions of order $(\omega R_i)^4$ and $(\omega {R_i}^2/\Delta)^4$ to the matching conditions that we neglect. Hence, the constant C will be chosen to match to the regions $\mathbf{I_i}$ and **III** appropriately. Furthermore, we can take the homogeneous parts of ϕ_1^{II} and ϕ_2^{II} to vanish and change the solutions in regions $\mathbf{I_i}$ and **III** accordingly. This means that we can consider the limits $\eta\to 1$, $\mu\to\pm1$ and the limit $\eta\gg 1$ in region **II** and solve there for the inhomogeneous parts of ϕ_1^{II} and ϕ_2^{II}. Start by considering the limits $\eta\to 1$, $\mu\to\pm1$. Equations (3.27) and (3.28) become

$$\frac{1}{{\zeta_i}^5}\,\partial_{\zeta_i}\left({\zeta_i}^5\partial_{\zeta_i}\right)\phi_1^{II_i}=-\left(1+\left(\frac{R_j}{2\Delta}\right)^4+\left(\frac{R_i}{\Delta\zeta_i}\right)^4\right)C\,, \qquad (3.29)$$

$$\frac{1}{{\zeta_i}^5}\,\partial_{\zeta_i}\left({\zeta_i}^5\partial_{\zeta_i}\right)\phi_2^{II_i}=-\left(1+\left(\frac{R_j}{2\Delta}\right)^4+\left(\frac{R_i}{\Delta\zeta_i}\right)^4\right)\phi_1^{II_i}\,. \qquad (3.30)$$

Solving for these equations we obtain the asymptotic behavior of ϕ^{II} as $\eta\to 1$, $\mu\to\pm1$:

$$\phi^{II_i}\sim C\left[1-(\omega\Delta)^2\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\frac{{\zeta_i}^2}{12}+(\omega\Delta)^2\left(\frac{R_i}{\Delta}\right)^4\frac{1}{4{\zeta_i}^2}-\right.$$
$$\left.-\,(\omega\Delta)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\left(\frac{R_i}{\Delta}\right)^4\frac{1}{24}\log\zeta_i+\cdots\right]. \qquad (3.31)$$

Similarly, in the limit $\eta \gg 1$ eqs. (3.27) and (3.28) become

$$\frac{1}{\eta^5}\,\partial_\eta\left(\eta^5\partial_\eta\right)\phi_1^{II\infty} = -\left(1+\frac{{R_1}^4+{R_2}^4}{(\Delta\eta)^4}\right)C\,, \tag{3.32}$$

$$\frac{1}{\eta^5}\,\partial_\eta\left(\eta^5\partial_\eta\right)\phi_2^{II\infty} = -\left(1+\frac{{R_1}^4+{R_2}^4}{(\Delta\eta)^4}\right)\phi_1^{II\infty} \tag{3.33}$$

which gives the following asymptotic behavior of ϕ^{II} for $\eta \gg 1$:

$$\begin{aligned}\phi^{II\infty} \sim C\Bigg[&1-(\omega\Delta)^2\,\frac{\eta^2}{12}+(\omega\Delta)^2\,\frac{{R_1}^4+{R_2}^4}{\Delta^4}\,\frac{1}{4\eta^2}-\\ &-(\omega\Delta)^4\,\frac{{R_1}^4+{R_2}^4}{\Delta^4}\,\frac{1}{24}\log\eta+\cdots\Bigg].\end{aligned} \tag{3.34}$$

Finally, consider the far region **III**. Instead of dropping the last term in eq. (3.13), we solve this equation by expanding $\phi^{III}(\rho)$ in powers of $\omega^4({R_1}^4+{R_2}^4)$:

$$\phi^{III}(\rho) = \phi_0^{III}(\rho)+\omega^4\left({R_1}^4+{R_2}^4\right)\phi_1^{III}(\rho)+\cdots. \tag{3.35}$$

Then, the analysis is entirely similar to the one performed for the near horizon regions $\mathbf{I_i}$, therefore we present just the final result

$$\begin{aligned}\frac{1}{D}\,\phi^{III}(\rho) = &\frac{J_2(\rho)}{\rho^2}-\omega^4\left({R_1}^4+{R_2}^4\right)\frac{\pi}{2\rho^2}\times\\ &\times\int^\rho \frac{d\sigma}{\sigma^3}\,J_2(\sigma)\left(J_2(\sigma)N_2(\rho)-N_2(\sigma)J_2(\rho)\right).\end{aligned} \tag{3.36}$$

For small ρ this gives

$$\frac{1}{D}\,\phi^{III}(\rho) \sim \frac{1}{8}\left(1-\frac{\rho^2}{12}\right)+\frac{\omega^4\left({R_1}^4+{R_2}^4\right)}{32\rho^2}\left(1-\frac{1}{6}\,\rho^2\log\rho\right)+\cdots. \tag{3.37}$$

Now we are in position to determine the improved matching conditions across the different regions. To match the expansions (3.23) and (3.31) for ϕ^{I_i} and ϕ^{II_i} we define the variable u_i by

$$u_i = \frac{R_i}{\Delta}\,\zeta_i\,,\qquad z_i = (\omega R_i)\left(\frac{R_i}{\Delta}\right)^2\frac{1}{u_i}\,, \tag{3.38}$$

where we assumed that $R_i \gg \Delta$. Notice that this condition is necessary in order to keep u_i finite of order unit in the domain of validity of the expansions for ϕ^{I_i} and ϕ^{II_i}. Replacing (3.38) in these expansions we have

$$\frac{1}{A_i}\,\phi^{I_i} \sim -\frac{4i}{\pi}\Bigg[1+(\omega R_i)^2\left(\frac{R_i}{\Delta}\right)^4\frac{1}{4{u_i}^2}-(\omega R_i)^2\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\left(\frac{\Delta}{R_i}\right)^4\frac{{u_i}^2}{12}+$$
$$+(\omega R_i)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\frac{1}{24}\left(\log\left(\frac{\omega {R_i}^2}{\Delta}\right)-\log\left(\frac{\Delta u_i}{R_i}\right)\right)+\cdots\Bigg],$$
$$\frac{1}{C}\,\phi^{II_i} \sim 1+(\omega R_i)^2\left(\frac{R_i}{\Delta}\right)^4\frac{1}{4{u_i}^2}-(\omega R_i)^2\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\left(\frac{\Delta}{R_i}\right)^4\frac{{u_i}^2}{12}-$$
$$-(\omega R_i)^4\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\frac{1}{24}\log\left(\frac{\Delta u_i}{R_i}\right)+\cdots. \tag{3.39}$$

By adjusting the constants A_i, we can match the logarithmic terms in the expansions:

$$A_i = 1-\frac{(\omega R_i)^4}{24}\left(1+\left(\frac{R_j}{2\Delta}\right)^4\right)\log\left(\frac{\omega {R_i}^2}{\Delta}\right), \qquad C=-\frac{4i}{\pi}. \tag{3.40}$$

The next corrections will be of order $(\omega R_i)^4$ and $(\omega R_i R_j/\Delta)^4$, but we neglect these. In the case $R_i \ll \Delta$, we define instead $u_i = \frac{\Delta}{R_i}\zeta_i$ which can be kept finite in the matching region. Of course the constants A_i and C will still be given by (3.40).

Similarly, to match the expansions (3.34) and (3.37) for ϕ^{II_∞} and ϕ^{III}, we define the variable u by

$$u=\frac{\Delta}{R_1+R_2}\,\eta\,, \qquad \rho=\omega\,(R_1+R_2)\,u\,, \tag{3.41}$$

where we assumed that $R_i \gg \Delta$. Across the matching region, u can be kept fixed of order unit and the expansions ϕ^{II_∞} and ϕ^{III} read

$$\frac{1}{C}\,\phi^{II_\infty} \sim 1-\omega^2\,(R_1+R_2)^2\,\frac{u^2}{12}+\omega^2\frac{{R_1}^4+{R_2}^4}{(R_1+R_2)^2}\,\frac{1}{4u^2}-$$
$$-\omega^4\left({R_1}^4+{R_2}^4\right)\frac{1}{24}\log\left(\frac{R_1+R_2}{\Delta}\,u\right)+\cdots,$$
$$\frac{1}{D}\,\phi^{III} \sim \frac{1}{8}\Bigg[1-\omega^2\,(R_1+R_2)^2\,\frac{u^2}{12}+\omega^2\frac{{R_1}^4+{R_2}^4}{(R_1+R_2)^2}\,\frac{1}{4u^2}-$$
$$-\omega^4\left({R_1}^4+{R_2}^4\right)\frac{1}{24}\log\left(\omega\left(R_1+R_2\right)u\right)\cdots\Bigg]. \tag{3.42}$$

By adjusting the constant D we can match the logarithmic terms in the expansions:

$$D=-\frac{32i}{\pi}\left[1+\frac{\omega^4}{24}\left({R_1}^4+{R_2}^4\right)\log\left(\omega\Delta\right)\right]. \tag{3.43}$$

In the case $R_i \ll \Delta$, we define instead $u=\frac{R_1+R_2}{\Delta}\eta$ which can be kept finite across the matching region.

With the improved matching conditions we can calculate the fluxes at the horizons and at infinity. Then, the corrected absorption probability by the i-th hole is seen to be

$$\mathcal{P}_i = \frac{\pi^2(\omega R_i)^8}{(16)^2}\left[1 - \frac{(\omega R_i)^4}{12}\left(1 + \left(\frac{R_j}{2\Delta}\right)^4\right)\log\left(\frac{\omega {R_i}^2}{\Delta}\right) - \right.$$
$$\left. - \frac{\omega^4\left({R_1}^4 + {R_2}^4\right)}{12}\log\left(\omega\Delta\right)\right] \tag{3.44}$$

which holds either for $R_i \gg \Delta$ or $R_i \ll \Delta$. This is our main result. In the reminder of this paper we interpret these corrections to the absorption probability from a field theory point of view.

As an aside, notice that this calculation can be easily extended to the multi-centered D3-brane geometry case. The dual field theory is the large-N_i SYM theory at strong coupling with gauge group broken to $\mathrm{S}(\prod_i \mathrm{U}(N_i))$. Then, the absorption probability by the i-th hole is

$$\mathcal{P}_i = \frac{\pi^2(\omega R_i)^8}{(16)^2}\left[1 - \frac{(\omega R_i)^4}{12}\left(1 + \sum_{j\neq i}\left(\frac{R_j}{\Delta_{ij}}\right)^4\right)\log\left(\frac{\omega {R_i}^2}{\Delta}\right) - \right.$$
$$\left. - \frac{\omega^4\sum_i {R_i}^4}{12}\log\left(\omega\Delta\right)\right], \tag{3.45}$$

where Δ_{ij} is the separation between the i-th and j-th D3-branes and Δ is a typical brane separation. The field theory analysis of this case is entirely similar to the double-centered case presented in the next section.

4. Field theory analysis

We are interested in maximally supersymmetric RG flows for $\mathcal{N} = 4$ SYM theory that arise from breaking the conformal invariance. In the infrared it has been conjectured that these flows can be viewed as a deformation of the $\mathcal{N} = 4$ superconformal fixed points by some irrelevant operators [18, 19, 24]. In particular, the gauge coupling beta function vanishes and the dimension of operators in short supersymmetry representations are constant along such flows.

First consider the D3-brane background with $\mathrm{SO}(6) \cong \mathrm{SU}(4)$ invariant harmonic function

$$H = h + \left(\frac{R}{r}\right)^4. \tag{4.1}$$

Interpreting r as the RG energy scale the dual field theory flows in the IR ($r \to 0$) to the $\mathcal{N} = 4$ superconformal fixed point. The corresponding deformation, which is controlled by the parameter h, can be seen to arise from the irrelevant Dirac-Born-Infeld (DBI) corrections to the SYM theory [18]. Alternatively, on the basis of

PSU$(2,2|4)$ representation theory the lagrangian of the dual field theory was conjectured to be [19, 24]

$$\mathcal{L} = \mathcal{L}_0 - \frac{h}{8T_3}\,\mathcal{O}_8\,, \tag{4.2}$$

where

$$\mathcal{L}_0 = -\frac{1}{4}\,\mathrm{Tr}\left(F_{AB}F^{AB}\right) \equiv -\frac{1}{4}\,\mathcal{O}_4 \tag{4.3}$$

is the usual bosonic SYM lagrangian and $\mathcal{O}_8$ is a dimension 8 operator preserving 16 supersymmetries and the SU$(4)_R$ symmetry. We use the ten-dimensional indices A, B so that F_{AB} is short for $F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$, $F_{\mu m} = D_\mu \phi_m$ with $D_\mu \equiv \partial_\mu + i g_{\mathrm{YM}}[A_\mu,\ ]$ and $F_{mn} = i g_{\mathrm{YM}}[\phi_m, \phi_n]$. Notice that we removed the gauge coupling from the front of the action by rescalling the fields according to $(A_\mu, \phi^m) \to g_{\mathrm{YM}}(A_\mu, \phi^m)$. The D3-brane tension T_3 is given by $T_3 = ((2\pi)^3 g_s \alpha'^2)^{-1}$.

In the case where there are two stacks of D3-branes separated by a distance 2Δ, the harmonic function (4.1) becomes

$$H = h + \frac{{R_1}^4}{|\vec{r} - \vec{\Delta}|^4} + \frac{{R_2}^4}{|\vec{r} + \vec{\Delta}|^4}\,. \tag{4.4}$$

The corresponding dual field theory is still given by (4.2) but the SU(N) gauge symmetry is broken to S$(\mathrm{U}(N_1) \times \mathrm{U}(N_2))$ by the non-zero expectation values for the scalar fields as described in the Introduction. For example, this means that there will be a mass term in the lagrangian $\mathcal{L}_0$ for the W particles.

Our main goal is to identify the origin and corresponding physical processes associated with the logarithmic corrections to the cross-section calculated in the previous section. For worldvolume on-shell processes involving the coupling of the dilaton field to the brane, the important terms in the lagrangian (4.2) that arise from the DBI action are

$$\begin{aligned}
\mathcal{O}_4 &= \mathrm{Tr}\left(F_{\mu\nu}\,F^{\mu\nu}\right), \\
\mathcal{O}_8 &= \mathrm{Tr}\left(F_{\mu\nu}\,F^{\nu\rho}\,F_{\rho\sigma}F^{\sigma\mu} - \frac{1}{4}\left(F_{\mu\nu}\,F^{\mu\nu}\right)^2\right).
\end{aligned} \tag{4.5}$$

Also, the coupling of the dilaton field to the brane world-volume is seen to be

$$S_{\mathrm{int}} = \int d^4x\ \phi\ \mathcal{O}_\phi = \int d^4x\ \phi\ \frac{1}{4}\left[\mathcal{O}_4 + \frac{h}{T_3}\,\mathcal{O}_8 + \cdots\right], \tag{4.6}$$

where $\mathcal{O}_4$ and $\mathcal{O}_8$ are defined in (4.5). Notice that a more careful analysis to account for numerical factors would require the use of the symmetrized trace as proposed by Tseytlin [31].

To calculated the cross-section one starts by calculating the two-point function of the operator $\mathcal{O}_\phi$

$$\Pi(x) = \left\langle \mathcal{O}_\phi(x)\mathcal{O}_\phi(0)\right\rangle_h\ , \tag{4.7}$$

where the subscript h reminds us that we are working with the deformed lagrangian (4.2). Then, the absorption cross-section is given by [32]

$$\sigma = \frac{2\kappa^2}{2i\omega} \text{Disc } \Pi(s)\Big|_{\substack{p^0=\omega \\ \vec{p}=\vec{0}}} , \tag{4.8}$$

where $s = -p^2$ and $\Pi(s)$ is the Fourier transform of $\Pi(x)$. The momentum space two-point function $\Pi(s)$ is analytic in the complex s-plane except possibly for poles or cuts on the real axis. The discontinuity across the cuts is given by Disc $\Pi(s) = \Pi(s + i\epsilon) - \Pi(s - i\epsilon)$.

Now that we have reviewed the connection between the classical cross-section and the dual field theory, we interpret the logarithmic corrections to the cross-section found earlier from the field theory point of view. We shall consider the two cases $R_i \gg \Delta$ and $R_i \ll \Delta$ discussed in section 2 separately.

4.1 Small brane separation ($R_i \gg \Delta$)

First, consider the case where Δ/α' is kept fixed in the $\alpha' \to 0$ decoupling limit. The gravitational dual geometry is sketched in figure 1c of section 2. In this case the absorption probability becomes

$$\mathcal{P}_i = \frac{\pi^2(\omega R_i)^8}{(16)^2}\left[1 - \frac{(\omega R_i)^4}{12}\left(\frac{R_j}{2\Delta}\right)^4 \log\left(\frac{\omega {R_i}^2}{\Delta}\right)\right]. \tag{4.9}$$

The tree-like throat geometry is dual to the $\mathcal{N} = 4$ SYM theory away from the origin of the moduli space of vacua, i.e. there is a VEV for the Higgs field that breaks conformal invariance and the gauge symmetry to $\mathrm{S}(\mathrm{U}(N_1) \times \mathrm{U}(N_2))$. The corresponding harmonic function in the gravity solution is given by (4.4) with $h = 0$. The theory flows from the UV conformal point with gauge group $\mathrm{SU}(N_1 + N_2)$ to the IR conformal fixed points with gauge group $\mathrm{SU}(N_i)$.

In the gravity calculation we assumed $\omega \ll m_W$, which means that we are studying deformations of the IR fixed points. In the case of the i-th throat, we are studying the RG flow near the $\mathrm{SU}(N_i)$ IR fixed point with conformal invariance broken due to a deformation by an irrelevant operator of dimension 8. In fact, down the throat the harmonic function becomes effectively [24]

$$H = \left(\frac{R_j}{2\Delta}\right)^4 + \left(\frac{R_i}{r}\right)^4 , \tag{4.10}$$

where $r \equiv \Delta\zeta_i$ is the radial coordinate centered at the i-th stack of branes. We conclude that in this limit the theory is described by the lagrangian (4.2) with $h = (R_j/2\Delta)^4$. The deformation in this lagrangian is the result of integrating out the massive W particles, defining an effective lagrangian near the IR fixed point. This is analogous to the situation where a brane is placed in the near-horizon geometry of a large number of branes. There, the correct action describing the probe dynamics is the DBI action, which effectively describes the effect of integrating out all the massive strings stretching between the probe and the branes [33]–[36].

To make the idea of the previous paragraph more precise consider the $\mathrm{SU}(N_1+N_2)$ SYM theory with bosonic lagrangian given by (4.3). The VEV for the scalars $g_{\mathrm{YM}}\langle\vec{\phi}\rangle = \frac{\vec{\Delta}}{2\pi\alpha'}\,\mathrm{Diag}(I_1,-I_2)$ breaks conformal invariance and the gauge symmetry to $\mathrm{S}(\mathrm{U}(N_1)\times\mathrm{U}(N_2))$. For energies $\epsilon \ll m_W$, we can integrate out the W particles and obtain an effective action for the $\mathrm{SU}(N_i)$ light degrees of freedom. This is similar to the case $N_1 = N$ and $N_2 = 1$ that has been extensively studied in the literature [33]–[36]. We refer the reader to these references for the details. Since we are interested in the large-N limit only the planar diagrams contribute to the effective potential. The first non-vanishing one-loop diagram involves 4 $\mathrm{SU}(N_i)$ coloured external legs. In our conventions, the resulting contribution to the lagrangian is [35]

$$\mathcal{L}_1 = -\frac{\pi^2 g_{\mathrm{YM}}^4}{(2\Delta/\alpha')^4}\,N_j\,\mathrm{Tr}\left(F_{AB}\,F^{BC}\,F_{CD}\,F^{DA} - \frac{1}{4}\left(F_{AB}\,F^{AB}\right)^2\right), \qquad (4.11)$$

where $g_{\mathrm{YM}}^2 = 2\pi g_s$ and the factor N_j is because we have a $\mathrm{SU}(N_j)$ colour index around the loop to sum over. Notice that the fields in this expression take values in the $\mathrm{SU}(N_i)$ Lie algebra. We see that for $h = (R_j/2\Delta)^4 = (2\alpha'^2 g_{\mathrm{YM}}^2 N_j)/(2\Delta)^4$ the one-loop result (4.11) gives exactly the same answer as (4.2) if we take the irrelevant operator $\mathcal{O}_8$ to arise from the DBI corrections. Furthermore, the F^4 terms are exact because they are not renormalized by higher loop diagrams [37]. To summarize, we start with pure SYM theory and break conformal invariance by going to the Coulomb branch of the theory. By integrating out the massive W particles we derived an effective lagrangian near the IR fixed points that has the form conjectured in [18, 19, 24].

Following the analysis of [18] the two-point function for the operator $\mathcal{O}_\phi$ dual to the dilaton field is

$$\begin{aligned}
\left\langle \mathcal{O}_\phi(x)\mathcal{O}_\phi(0)\right\rangle_{h=(R_j/2\Delta)^4} &= \int \mathcal{D}A_\mu\, e^{-\int d^4z\left[\frac{1}{4}\mathcal{O}_4 + \frac{h}{8T_3}\mathcal{O}_8\right]}\,\mathcal{O}_\phi(x)\,\mathcal{O}_\phi(0) = \\
&= \left\langle \mathcal{O}_\phi(x)\,\mathcal{O}_\phi(0)\left(1 - \left(\frac{R_j}{2\Delta}\right)^4 \frac{1}{8T_3}\int d^4z\,\mathcal{O}_8(z)\right)\right\rangle_{h=0} = \\
&= \frac{1}{2^4}\left\langle \mathcal{O}_4(x)\mathcal{O}_4(0)\right\rangle_{h=0} - \\
&\quad - \left(\frac{R_j}{2\Delta}\right)^4 \frac{1}{2^7 T_3}\int d^4z\left\langle \mathcal{O}_4(x)\,\mathcal{O}_8(z)\,\mathcal{O}_4(0)\right\rangle_{h=0}, \qquad (4.12)
\end{aligned}$$

where we are just keeping the term that will give the *log* correction calculated in the gravity approach. Notice that the two-point function for the operator $\mathcal{O}_\phi$ is written in terms of two- and three-point functions of chiral primary operators at the IR

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$SU(N_i)$ conformal fixed point. Since these are expected to be protected [32], [38]–[43] the perturbative field theory and strongly coupled gravity approaches should give the same result. Now the analysis is entirely similar to the one presented in [18]. We refer the reader to that paper for the details. The first term in the last equality gives the leading term in the scattering cross-section, while the second term gives the logarithmic correction. The latter is associated with a process where a virtual pair of gauge particles interacts with the incident wave to create a pair of gauge particles. The strength of such interaction is determined by the effective action that arised from integrating out the W particles. To match the logarithmic term in the field theory calculation we should set the ultraviolet cut off to $\Lambda = \Delta/{R_i}^2$. This is related to the natural scale in the field theory side m_W by $\Lambda \sim m_W/\sqrt{gN_i}$. A better understanding of the ultraviolet cut off requires extending the cross-section calculation to the next order [18]. Notice, however, that the scale $\Lambda = \Delta/{R_i}^2$ is the gravity mass gap and may be related to the existence of colour singlet condensates of W particles at strong coupling [15, 25].

4.2 Large brane separation ($R_i \ll \Delta$)

As explained in section 2 the two throats can be separated in the decoupling limit as long as $\Delta/\alpha' \to \infty$ (recall figures $1c$ and $1d$). Here we consider the effects of connecting both throats using the full D3-brane geometry with harmonic function given by (4.4) with $h = 1$. In this case the absorption probability becomes

$$\mathcal{P}_i = \frac{\pi^2(\omega R_i)^8}{(16)^2}\left[1 - \frac{(\omega R_i)^4}{6}\log(\omega R_i) - \frac{(\omega R_j)^4}{12}\log(\omega\Delta)\right]. \tag{4.13}$$

In the dual field theory we start with the action (4.2) with gauge group $SU(N_1+N_2)$ and $h = 1$. Additionally, the VEV for the Higgs field breaks the gauge symmetry to $S(U(N_1)\times U(N_2))$. In other words, we consider the Coulomb branch of the deformed theory. This is very similar to the single-centered case studied in [18] where conformal invariance was broken by setting $h = 1$ while remaining at the origin of the moduli space of vacua. Intriligator argued that in the UV this theory is dual to the asymptotically flat space. In the IR we end up with the $SU(N_i)$ IR conformal fixed points corresponding to each throat.

It is convenient to write the operators $\mathcal{O}_4$ and $\mathcal{O}_8$ in (4.5) using the following notation

$$\begin{aligned}
\mathcal{O}_4 &\equiv \mathcal{O}_4^{\gamma_1} + \mathcal{O}_4^{W} + \mathcal{O}_4^{\gamma_2} \sim \sum_{a,b} F^{ab}F^{ba} + 2\sum_{a,\bar b} F^{a\bar b}F^{\bar b a} + \sum_{\bar a,\bar b} F^{\bar a\bar b}F^{\bar b\bar a}\,,\\
\mathcal{O}_8 &\equiv \mathcal{O}_8^{\gamma_1} + \mathcal{O}_8^{\gamma_1 W} + \mathcal{O}_8^{W} + \cdots \sim\\
&\sim \sum_{a,b,c,d} F^{ab}F^{bc}F^{cd}F^{da} + 4\sum_{\bar a,b,c,d} F^{\bar a b}F^{bc}F^{cd}F^{d\bar a} +\\
&\quad + 2\sum_{\bar a,b,\bar c,d} F^{\bar a b}F^{b\bar c}F^{\bar c d}F^{d\bar a} + \cdots,
\end{aligned} \tag{4.14}$$

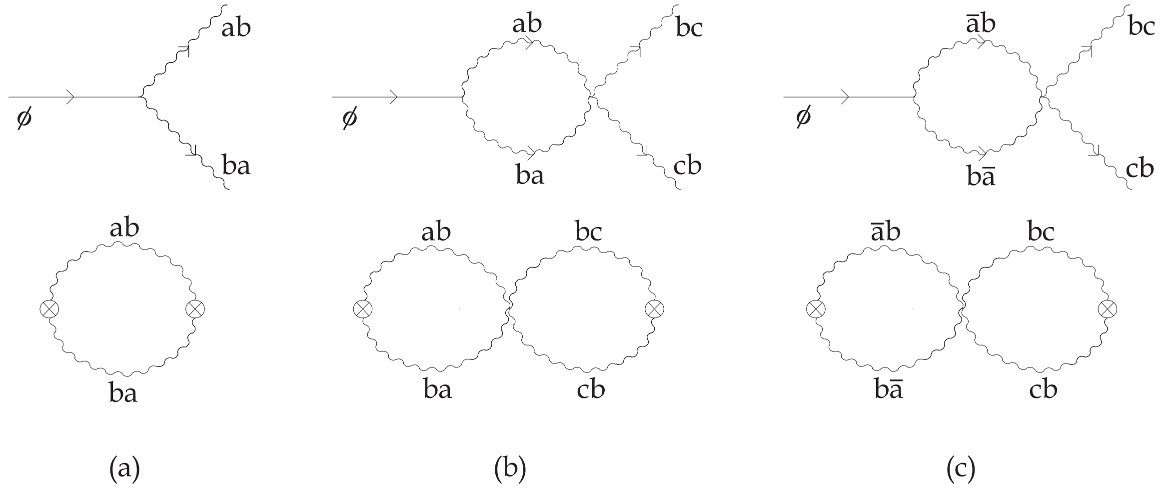

Figure 3: Feynman diagrams for the absorption cross-section by the first black hole. After summing over the YM indices we see that the leading order term arising from the diagram (a) is proportional to $(N_1)^2$ and that the logarithmic corrections that arise from the closed loops in the top diagrams of (b) and (c) are proportional to $(N_1)^3$ and $(N_1)^2N_2$, respectively. This is in agreement with the gravity calculation.

where the indices $a, b, \ldots$ run over the $\mathrm{SU}(N_1)$ gauge group and the indices $\bar{a}, \bar{b}, \ldots$ over the $\mathrm{SU}(N_2)$ gauge group. In the expression for $\mathcal{O}_8$ we wrote just the terms that are important for the absorption by the first hole. Also, we dropped a four point vertex with one $\mathrm{SU}(N_1)$ gauge particle, one $\mathrm{SU}(N_2)$ gauge particle and two W bosons ($\sim F^{\bar{a}\bar{b}}F^{\bar{b}c}F^{cd}F^{d\bar{a}}$) because it gives a negligible correction for large N. Similarly to the previous section, the two-point function for the operator $\mathcal{O}_\phi$ is

$$\begin{aligned}
\left\langle \mathcal{O}_\phi(x)\mathcal{O}_\phi(0)\right\rangle_{h=1} &= \int \mathcal{D}A_\mu\, e^{-\int d^4z\left[\frac{1}{4}\mathcal{O}_4+\frac{1}{8T_3}\mathcal{O}_8\right]}\,\mathcal{O}_\phi(x)\,\mathcal{O}_\phi(0) = \\
&= \left\langle \mathcal{O}_\phi(x)\,\mathcal{O}_\phi(0)\left(1-\frac{1}{8T_3}\int d^4z\,\mathcal{O}_8(z)\right)\right\rangle_{h=0} = \\
&= \sum_{i=1}^{2}\left[\frac{1}{2^4}\left\langle \mathcal{O}_4^{\gamma_i}(x)\,\mathcal{O}_4^{\gamma_i}(0)\right\rangle_{h=0} - \right. \\
&\qquad - \frac{1}{2^7T_3}\int d^4z\left\langle \mathcal{O}_4^{\gamma_i}(x)\,\mathcal{O}_8^{\gamma_i}(z)\,\mathcal{O}_4^{\gamma_i}(0)\right\rangle_{h=0} - \\
&\qquad \left. - \frac{1}{2^6T_3}\int d^4z\left\langle \mathcal{O}_4^{W}(x)\,\mathcal{O}_8^{\gamma_i W}(z)\,\mathcal{O}_4^{\gamma_i}(0)\right\rangle_{h=0}\right] + \\
&\quad + \frac{1}{2^4}\left\langle \mathcal{O}_4^{W}(x)\,\mathcal{O}_4^{W}(0)\right\rangle_{h=0} - \\
&\quad - \frac{1}{2^7T_3}\int d^4z\left\langle \mathcal{O}_4^{W}(x)\,\mathcal{O}_8^{W}(z)\,\mathcal{O}_4^{W}(0)\right\rangle_{h=0}, \qquad (4.15)
\end{aligned}$$

where we are just keeping the leading terms. First consider the terms inside the squared brackets in the last equality. The first term is the two-point function for the

SU(N_i) theory at the infrared conformal fixed point and it gives rise to the leading term in the absorption probability (4.13). This term is associated with a process where the incident wave interacts with the brane to create a pair of gauge particles (see figure 3a) [30]. The first logarithmic correction in the absorption probability (4.13) arises from the second term in the squared brackets. This correction was analyzed in [18] and it is associated with a process where a virtual pair of gauge particles interacts with the incident dilaton wave to create a pair of gauge particles (see figure 3b). Finally, the other logarithmic correction in the cross-section arises from the third term in the squared brackets. This corresponds to a process where a virtual pair of W bosons interacts with the incident dilaton wave to create a pair of gauge particles (see figure 3c). The last two field theory processes have the correct energy and N_i dependence to account for the logarithmic corrections to the cross-section. However, how to match precisely the numerical factors remains an open question. As observed in the previous subsection, an appropriate understanding of the UV cut off would require extending the analysis to the next order.

The cross-section for all the above processes is calculated from the discontinuity of the momentum space two-point function across the positive real axis in the s-plane. In fact, the correction $\int d^4z\langle \mathcal{O}_4^W(x)\mathcal{O}_8^{\gamma_i W}(z)\mathcal{O}_4^{\gamma_i}(0)\rangle_{h=0}$ is also associated to the creation of a pair of W bosons via a virtual pair of gauge particles. In this case the momentum space two-point function has an extra discontinuity in the s-plane over the real axis for $s \geq m_W$. Since we work at energies $\omega \ll m_W$, well below the threshold where the channel for W boson pair production opens, this process is not seen in the gravity calculation here employed. Similarly, the Fourier transform of the terms outside the squared brackets in (4.15) will have a cut for $s \geq m_W$ and are associated with the creation of W boson pairs.

5. Concluding remarks

The main goal of this work was to investigate the Coulomb branch of $\mathcal{N} = 4$ SYM theory using the AdS/CFT correspondence. This was done by studying the scattering of a minimally coupled scalar field by two parallel stacks of D3-branes. Technical reasons forced us to consider processes at energy scales much smaller than the threshold for W boson pair production. Despite this fact we were still able to find new phenomena in this phase of the theory. Firstly, for small brane separation, we considered the field theory dual to the tree-like throat geometry. This is a point on the Coulomb branch of $\mathcal{N} = 4$ SYM theory. Near the IR fixed points the theory can be described by integrating out the massive W particles. As a result, the effective lagrangian leads to a correction to the cross-section that is associated with the creation of gauge particles through the production of virtual pairs of gauge particles. This result is perfectly consistent with the classical gravity calculation. Secondly, for large brane separation, we considered the theory that results from connecting

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both throats with asymptotic flat space. This is associated with the DBI corrections to the SYM theory. In this case we found corrections to the cross-section that are associated with the creation of gauge particle pairs through the production of virtual pairs of gauge particles and of W bosons. For both small or large brane separation the W particles that arise from the $\mathrm{SU}(N) \to \mathrm{S}(\mathrm{U}(N_1) \times \mathrm{SU}(N_2))$ symmetry breaking process are seen to play an important role in the gravity/gauge theory correspondence. While these particles are essential to identify the appropriate field theory processes associated with the logarithmic corrections to the classical cross-section, the problem of matching exactly the numerical factors remains unsolved.

It would be very interesting to extend our results to higher energies. In particular, one would like to see the channel for W bosons pair production to open up in the gravitational side of the correspondence. However, the scale in the gravity side is given by the mass gap energy $m_g = m_W/\sqrt{gN}$. As noticed earlier, this fact is expected to be related to the existence of colour singlets of BPS stretched strings with large binding energy. Hence, by studying the classical absorption for energies $\epsilon \sim m_g$ we could obtain some information about these bound states. Maybe our best hope would be to study numerically the solutions of Laplace equation in the double-centered D3-brane geometry. As far as the W particles are concerned, it looks that in the gravity limit these particles are not seen directly and that we may need the full string theory on $AdS_5 \times S^5$ to recover them.

An obvious extension of this work is to consider the scalar absorption by the double centered M-brane geometries. Unfortunately, in this case we do not have the input from D-brane and SYM physics to fully understand the dual field theory picture. Another interesting venue of research would be to study the four- and five-dimensional black hole cases. These are related to the $\mathrm{AdS}_2/\mathrm{CFT}_1$ and $\mathrm{AdS}_3/\mathrm{CFT}_2$ dualities, respectively.

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